Cartesian Product in Neutrosophic Fuzzy Ring

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Abstract: **-**Neutrosophic fuzzy set is an extension of fuzzy set and intusionistic fuzzy set. The algebraic properties of Neutrosophic fuzzy ideals of ring is dispensed in this paper. Further Cartesian product of neutrosophic fuzzy set is defined and examined its ideals.

Keywords: *Neutrosophic fuzzy set, ring, neutrosophic fuzzy ring, neutrosophic fuzzy ideals of ring.*

1. Introduction

In 1965, Zadeh [1] proposed the notion of fuzzy set. Fuzzy group developed by A. Rosenfeld [2]. Many authors in many directions digged the research in fuzzy sets in many directions. Various algebraic structures like groups, rings, γ rings, γ near rings, semi rings, hyper rings etc., developed in fuzzy set. Further these algebraic structures developing in neutrosophic fuzzy sets has been started. Neutrosophic sets which is a field of philosophy where indeterminacy included. Neutrosophic theory developed by F. Smarandache [7,8] in 1995. Usually, neutrosophic set theory is estimated by three components T, I, F. The neutrosophic set theory have seen great triumph in several fields such as image processing, medical diagnosis, decision making and so on. Then onwards, authors like I Arockarani [15] extended the theory of neutrosophic fuzzy set. K. Hemabala et.al. [9,11,12,13,14] developed theorems in γ near rings, anti neutrosophic fuzzy theorems in γ near rings and medical applications in refined neutrosophic fuzzy logic. Rajan Kumar et.al.[3], Liu[4], Malik [5], Kumbhojkar et.al.[6] developed concepts in fuzzy rings and groups. Further, Agboola A.A.A [10] established neutrosophic groups and sub groups. Based on this literature, we give brief concepts in neutrosophic fuzzy rings and verified some algebraic properties.

2. Preliminaries:

Basic definition of fuzzy set, neutrosophic fuzzy set and fuzzy ring are presented this section.

2.1 Definition: [1]

Let X be a non empty set and μ_v be a fuzzy set defined on X then $\mu_v = X \rightarrow [0, 1]$ such that

$\mu_v = \{\mu_v(x) / x \in X\}$

2.2 Definition [7,8]

Let X be a non empty set then neutrosophic fuzzy set F on X is defined as

$F = \{x ; F_t(x), F_i(x), F_f(x) : x \in X\}$

Where $F_t(x) : X \to [0, 1]$, $F_i(x) : X \to [0, 1]$, $F_f(x) : X \to [0, 1]$.

Where $F_t(x)$ is truth membership function

$F_t(x)$ is indeterminacy

$F_f(x)$ is false indeterminacy and $0^- \leq F_t(x)+ F_i(x)+F_f(x) \leq 3^+$

2.3 Definition [7,8]

Let F, G are two neutrosophic fuzzy sets, then we have the following relations and operations.

1. $F \leq G$ iff $F_t \leq G_t$, $F_i \geq G_i$, $F_j \geq G_j$
2. $F = G$ iff $F_t = G_t$, $F_i = G_i$, $F_f = G_f$
3. $F \cup G = \{x, \max(F_t(x), G_t(x)), \min(F_i(x), G_i(x)), \min(F_f(x), G_f(x)), x \in X\}$
4. $F \cap G = \{x, \min(F_t(x), G_t(x)), \max(F_i(x), G_i(x)), \max(F_f(x), G_f(x)), x \in X\}$

2.4 Definition [3]

A fuzzy set μ_v on a ring R is said to be a fuzzy ring R, all x, y ϵ R such that

i) $\mu_v(x - y) \geq \min\{\mu_v(x), \mu_v(y)\}$

ii) $\mu_v(xy) \geq \min\{\mu_v(x), \mu_v(y)\}$

2.5 Definition [3]

A fuzzy ring μ_v on R is said to be

i) Fuzzy left ideal, if $\mu_v(xy) \geq \mu_v(y)$ for all x, y ϵ R

ii) Fuzzy right ideal, if $\mu_v(xy) \geq \mu_v(x)$ for all x, y ϵ R

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2.6 Definition [3]

A fuzzy ring μ_v on a ring R is called a fuzzy ideal if it is both a fuzzy left ideal and a fuzzy right ideal.

3 Neutrosophic fuzzy set of ring

In this section we introduced the concept of neutrosophic fuzzy set of ring and developed some related theorems.

3.1 Definition

A neutosophic fuzzy set F on a ring R is said to be neutrosophic fuzzy sub ring of R if

i) $F_t(x - y) \geq \min\{F_t(x), F_t(y)\}$

$F_i(x - y) \leq \max\{F_i(x), F_i(y)\}$

$F_f(x - y) \leq \max\{F_f(x), F_f(y)\}$

ii) $F_t(xy) \geq \min\{F_t(x), F_t(y)\}$

$F_i(xy) \leq \max\{F_i(x), F_i(y)\}$

$F_f(xy) \leq \max\{F_f(x), F_f(y)\}$

3.2 Definition

A neutrosophic fuzzy ring F on R is said to neutrosophic

i) Fuzzy left ideal if

$F_t(xy) \geq F_t(y)$ for all x, y ϵ R

$F_i(xy) \leq F_i(y)$

$F_f(xy) \leq F_f(y)$

ii) Fuzzy right ideal if

$F_t(xy) \geq F_t(x)$

$F_i(xy) \leq F_i(x)$

$F_f(xy) \leq F_f(x)$

3.2 Definition

A neutrosophic fuzzy set F on R is called a neutrosophic fuzzy ideal if it is a both neutrosophic fuzzy left and right ideal of R.

3.3 Theorem

Let F and G are neutrosophic fuzzy sets. If F, G are neutrosophic left fuzzy ideal of R then $F \cup G$ is also neutrosophic left fuzzy ideal of R.

Proof:

i) $(F_t \cup G_t)(x-y) = \max\{F_t(x-y), G_t(x-y)\}$
$\geq \max\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \max \min\{(F_t(x), F_t(y), G_t(x), G_t(y)\}$
$\geq \min \max\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\max(F_t(x), G_t(x)), \max(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cup G_t)(x), (F_t \cup G_t)(y)\}$
$(F_i \cup G_i)(x-y) = \min\{F_i(x-y), G_i(x-y)\}$
$\leq \min\{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \min \max\{(F_i(x), F_i(y), G_i(x), G_i(y)\}$
$\leq \max\{\min(F_i(x), G_i(x)), \min(F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cup G_i)(x), (F_i \cup G_i)(y)\}$
$(F_f \cup G_f)(x-y) = \min\{F_f(x-y), G_f(x-y)\}$
$\leq \min\{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \min \max\{(F_f(x), F_f(y), G_f(x), G_f(y)\}$
$\leq \max\{\min(F_f(x), G_f(x)), \min(F_f(y)), G_f(y))\}$
$\leq \max\{\cup(F_f \cup G_f)(x), (F_f \cup G_f)(y)\}$

ii) $(F_t \cup G_t)(xy) = \max\{F_t(xy), G_t(xy)\}$
$\geq \max\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \max \min\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\max(F_t(x), G_t(x)), \max(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cup G_t)(x), (F_t \cup G_t)(y)\}$
$(F_i \cup G_i)(xy) = \min\{F_i(xy), G_i(xy)\}$
$\leq \min\{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \min \max\{(F_i(x), G_i(x)), (F_i(y), G_i(y))\}$
$\leq \max\{\min(F_i(x), G_i(x)), \min(F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cup G_i)(x), (F_i \cup G_i)(y)\}$
$(F_f \cup G_f)(xy) = \min\{F_f(xy), G_f(xy)\}$
$\leq \min\{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \min \max\{(F_f(x), G_f(x)), (F_f(y), G_f(y))\}$
$\leq \max\{\min(F_f(x), G_f(x)), \min(F_f(y)), G_f(y))\}$
$\leq \max\{(F_f \cup G_f)(x), (F_f \cup G_f)(y)\}$

iii) $(F_t \cup G_t)(xy) = \max\{F_t(xy), G_t(xy)\}$
$\geq \max[F_t(y), G_t(y)]$
$\geq (F_t \cup G_t)(y)$

$(F_i \cup G_i)(xy) = \min\{F_i(xy), G_i(xy)\}$
$\leq \min\{F_i(y), G_i(y)\}$
$\leq (F_i \cup G_i)(y)$

$(F_f \cup G_f)(xy) = \min\{F_f(xy), G_f(xy)\}$
$\leq \min\{F_f(y), G_f(y)\}$
$\leq (F_f \cup G_f)(y)$

Hence, F ∪ G is a neutrosophic fuzzy left ideal of R.

3.4 Theorem

Let F and G are neutrosophic fuzzy sets. If F, G are neutrosophic right fuzzy ideal of R then F ∪ G is also neutrosophic right fuzzy ideal of R.

Proof:

i) $(F_t \cup G_t)(x - y) = \max\{F_t(x - y), G_t(x - y)\}$
$\geq \max\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \max \min\{(F_t(x), F_t(y), G_t(x), G_t(y)\}$
$\geq \min \max\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\max(F_t(x), G_t(x)), \max(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cup G_t)(x), (F_t \cup G_t)(y)\}$
$(F_i \cup G_i)(x - y) = \min\{F_i(x - y), G_i(x - y)\}$
$\leq \min\{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \min \max\{(F_i(x), F_i(y), G_i(x), G_i(y)\}$
$\leq \max\{\min(F_i(x), G_i(x)), \min(F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cup G_i)(x), (F_i \cup G_i)(y)\}$
$(F_f \cup G_f)(x - y) = \min\{F_f(x - y), G_f(x - y)\}$
$\leq \min\{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \min \max\{(F_f(x), F_f(y), G_f(x), G_f(y)\}$
$\leq \max\{\min(F_f(x), G_f(x)), \min(F_f(y)), G_f(y))\}$
$\leq \max\{\cup(F_f \cup G_f)(x), (F_f \cup G_f)(y)\}$

ii) $(F_t \cup G_t)(xy) = \max\{F_t(xy), G_t(xy)\}$
$\geq \max\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \max \min\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\max(F_t(x), G_t(x)), \max(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cup G_t)(x), (F_t \cup G_t)(y)\}$
$(F_i \cup G_i)(xy) = \min\{F_i(xy), G_i(xy)\}$
$\leq \min\{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \min \max\{(F_i(x), G_i(x)), (F_i(y), G_i(y))\}$
$\leq \max\{\min(F_i(x), G_i(x)), \min(F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cup G_i)(x), (F_i \cup G_i)(y)\}$
$(F_f \cup G_f)(xy) = \min\{F_f(xy), G_f(xy)\}$
$\leq \min\{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \min \max\{(F_f(x), G_f(x)), (F_f(y), G_f(y))\}$
$\leq \max\{\min(F_f(x), G_f(x)), \min(F_f(y)), G_f(y))\}$
$\leq \max\{(F_f \cup G_f)(x), (F_f \cup G_f)(y)\}$

iii) $(F_t \cup G_t)(xy) = \max\{F_t(xy), G_t(xy)\}$
$\geq \max[F_t(x), G_t(x)]$
$\geq (F_t \cup G_t)(x)$

$(F_i \cup G_i)(xy) = \min\{F_i(xy), G_i(xy)\}$
$\leq \min\{F_i(x), G_i(x)\}$
$\leq (F_i \cup G_i)(x)$

$(F_f \cup G_f)(xy) = \min\{F_f(xy), G_f(xy)\}$

$\leq \min\{F_f(x), G_f(x)\}$
$\leq (F_f \cup G_f)(x)$

Hence, F ∪ G is a neutrosophic fuzzy right ideal of R.

3.5 Theorem

Let F and G are neutrosophic fuzzy sets. If F, G are neutrosophic left fuzzy ideal of R then F ∩G is also neutrosophic left fuzzy ideal of R.

Proof:

i) $(F_t \cap G_t)(x-y) = \min\{F_t(x-y), G_t(x-y)\}$
$\geq \min\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \min \min\{(F_t(x), F_t(y), G_t(x), G_t(y)\}$
$\geq \min \min\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\min(F_t(x), G_t(x)), \min(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cap G_t)(x), (F_t \cap G_t)(y)\}$

$(F_i \cap G_i)(x-y) = \max\{F_i(x-y), G_i(x-y)\}$
$\leq \max\{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \max \max\{(F_i(x), F_i(y), G_i(x), G_i(y)\}$
$\leq \max\{\max(F_i(x), G_i(x)), \max(F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cap G_i)(x), (F_i \cap G_i)(y)\}$
$(F_f \cap G_f)(x-y) = \max\{F_f(x-y), G_f(x-y)\}$
$\leq \max\{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \max \max\{(F_f(x), F_f(y), G_f(x), G_f(y)\}$
$\leq \max\{\max(F_f(x), G_f(x)), \max(F_f(y)), G_f(y))\}$
$\leq \max\{\cup(F_f \cap G_f)(x), (F_f \cap G_f)(y)\}$

ii) $(F_t \cap G_t)(xy) = \min\{F_t(xy), G_t(xy)\}$
$\geq \min\{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \min \min\{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{\min(F_t(x), G_t(x)), \min(F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cap G_t)(x), (F_t \cap G_t)(y)\}$
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$(F_i \cap G_i)(xy) = \max\{F_i(xy), G_i(xy)\}$
$\leq \max \{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \max \max\{(F_i(x), G_i(x)), (F_i(y), G_i(y))\}$
$\leq \max\{ \max (F_i(x), G_i(x)), \max (F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cap G_i)(x), (F_i \cap G_i)(y)\}$

$(F_f \cap G_f)(xy) = \max \{F_f(xy), G_f(xy)\}$
$\leq \max \{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \max \max\{(F_f(x), G_f(x)), (F_f(y), G_f(y))\}$
$\leq \max\{ \max (F_f(x), G_f(x)), \max (F_f(y)), G_f(y))\}$
$\leq \max\{(F_f \cap G_f)(x), (F_f \cap G_f)(y)\}$

iii) $(F_t \cap G_t)(xy) = \min\{F_t(xy), G_t(xy)\}$
$\geq \min\{F_t(y), G_t(y)\}$
$\geq (F_t \cap G_t)(y)$

$(F_i \cap G_i)(xy) = \max \{F_i(xy), G_i(xy)\}$
$\leq \max \{F_i(y), G_i(y)\}$

$\leq (F_i \cap G_i)(y)$

$(F_f \cap G_f)(xy) \quad = \max \{F_f(xy), G_f(xy)\}$
$\leq \max \{F_f(y), G_f(y)\}$
$\leq (F_f \cap G_f)(y)$

Hence, F ∩ G is a neutrosophic fuzzy left ideal of R.

3.6 Theorem

Let F and G are neutrosophic fuzzy sets. If F, G are neutrosophic right fuzzy ideal of R then F ∩ G is also neutrosophic right fuzzy ideal of R.

Proof:

i) $(F_t \cap G_t)(x - y) \quad = \min \{F_t(x - y), G_t(x - y)\}$
$\geq \min \{\min(F_t(x), F_t(y)), \min(G_t(x), G_t(y))\}$
$\geq \min \min\{(F_t(x), F_t(y), G_t(x), G_t(y)\}$
$\geq \min \min \{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min\{ \min (F_t(x), G_t(x)), \min (F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cap G_t)(x), (F_t \cap G_t)(y)\}$
$(F_i \cap G_i)(x - y) \quad = \max \{F_i(x - y), G_i(x - y)\}$
$\leq \max \{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \max \max\{(F_i(x), F_i(y), G_i(x), G_i(y)\}$
$\leq \max\{ \max (F_i(x), G_i(x)), \max (F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cap G_i)(x), (F_i \cap G_i)(y)\}$
$(F_f \cap G_f)(x - y) \quad = \max \{F_f(x - y), G_f(x - y)\}$
$\leq \max \{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \max \max\{(F_f(x), F_f(y), G_f(x), G_f(y)\}$
$\leq \max\{\max(F_f(x), G_f(x)), \max (F_f(y)), G_f(y))\}$
$\leq \max\{\cup(F_f \cap G_f)(x), (F_f \cap G_f)(y)\}$

ii) $(F_t \cap G_t)(xy) \quad = \min\{F_t(xy), G_t(xy)\}$
$\geq \min \{ \min (F_t(x), F_t(y)), \min (G_t(x), G_t(y))\}$
$\geq \min \min \{(F_t(x), G_t(x)), (F_t(y)), G_t(y))\}$
$\geq \min \{ \min (F_t(x), G_t(x)), \min (F_t(y)), G_t(y))\}$
$\geq \min\{(F_t \cap G_t)(x), (F_t \cap G_t)(y)\}$

$(F_i \cap G_i)(xy) \quad = \max \{F_i(xy), G_i(xy)\}$
$\leq \max \{\max(F_i(x), F_i(y)), \max(G_i(x), G_i(y))\}$
$\leq \max \max\{(F_i(x), G_i(x)), (F_i(y), G_i(y))\}$
$\leq \max\{ \max (F_i(x), G_i(x)), \max (F_i(y)), G_i(y))\}$
$\leq \max\{(F_i \cap G_i)(x), (F_i \cap G_i)(y)\}$

$(F_f \cap G_f)(xy) \quad = \max \{F_f(xy), G_f(xy)\}$
$\leq \max \{\max(F_f(x), F_f(y)), \max(G_f(x), G_f(y))\}$
$\leq \max \max\{(F_f(x), G_f(x)), (F_f(y), G_f(y))\}$
$\leq \max\{ \max (F_f(x), G_f(x)), \max (F_f(y)), G_f(y))\}$
$\leq \max\{(F_f \cap G_f)(x), (F_f \cap G_f)(y)\}$

iii) $(F_t \cap G_t)(xy) \quad = \min\{F_t(xy), G_t(xy)\}$
$\geq \min\{F_t(x), G_t(x)\}$
$\geq (F_t \cap G_t)(x)$

$(F_i \cap G_i)(xy) \quad = \max\{F_i(xy), G_i(xy)\}$

$\leq$ max$\{F_i(x), G_i(x)\}$
$\leq (F_i \cap G_i)(x)$

$(F_f \cap G_f)(xy)$ = max$\{F_f(xy), G_f(xy)\}$
$\leq$ max$\{F_f(x), G_f(x)\}$
$\leq (F_f \cap G_f)(x)$

Hence, F ∩ G is a neutrosophic fuzzy right ideal of R.

3.7 Definition

Let F and G are two neutrosophic fuzzy sets in R & S. Then two Cartesian product of F & G is defined by
$F \times G = \{(x,y);\ \min(F_t(x), G_t(x)), \max(F_i(x), G_i(x)), \max(F_f(x), G_f(y)) \ / \ (x,y) \in R \times S\}$

3.8 Theorem

If F & G are two neutrosophic fuzzy left & right ideals of R & S then F x G is also neutrosophic fuzzy left and right ideals of R x S

Proof:

i) $(F_t \times G_t)((x_1,x_2) - (y_1,y_2)) = (F_t \times G_t)(x_1 - y_1 , x_2 - y_2)$
$= \min \{F_t(x_1 - y_1), G_t(x_2 - y_2)\}$
$\geq \min \{\min(F_t(x_1), F_t(y_1)), \min(G_t(x_2), G_t(y_2))\}$
$\geq \min \min \{(F_t(x_1), G_t(x_2)), (F_t(y_1)), G_t(y_2))\}$
$\geq \min\{ \min (F_t(x_1), G_t(x_2)), \min (F_t(y_1)), G_t(y_2))\}$
$\geq \min\{(F_t \times G_t)(x_1,x_2), (F_t \times G_t)(y_1,y_2)\}$
$(F_i \times G_i) ((x_1,x_2) - (y_1,y_2)) = (F_i \times G_i)(x_1 - y_1 , x_2 - y_2)$
$= \max \{F_i(x_1 - y_1), G_i(x_2 - y_2)\}$
$\leq \max \{\max (F_i(x_1), F_i(y_1)), \max(G_i(x_2), G_i(y_2))\}$
$\leq \max \max\{(F_i(x_1), F_i(y_1), G_i(x_2), G_i(y_2)\}$
$\leq \max \{\max (F_i(x_1), G_i(x_2)), \max(F_i(y_1)), G_i(y_2))\}$
$\leq \max \{(F_i \times G_i)(x_1,x_2), (F_i \times G_i)(y_1,y_2)\}$

$(F_f \times G_f) ((x_1,x_2) - (y_1,y_2)) = (F_f \times G_f)(x_1 - y_1 , x_2 - y_2)$
$= \max \{F_f(x_1 - y_1), G_f(x_2 - y_2)\}$
$\leq \max\{\max(F_f(x_1), F_f(y_1)), \max(G_f(x_2), G_f(y_2))\}$
$\leq \max \max\{(F_f(x_1), F_f(y_1), G_f(x_2), G_f(y_2)\}$
$\leq \max\{\max(F_f(x_1), G_f(x_2)), \max(F_f(y_1)), G_f(y_2))\}$
$\leq \max\{(F_f \times G_f)(x_1,x_2), (F_f \times G_f)(y_1,y_2)\}$
ii) $(F_t \times G_t)(x_1y_1, x_2y_2) = \min\{F_t(x_1y_1), G_t(x_2y_2)\}$
$\geq \min\{\min(F_t(x_1), F_t(y_1)), \min(G_t(x_2), G_t(y_2))\}$
$\geq \min \min\{(F_t(x_1), G_t(x_2)), (F_t(y_1)), G_t(y_2))\}$
$\geq \min\{\min(F_t(x_1), G_t(x_2)), \min(F_t(y_1)), G_t(y_2))\}$
$\geq \min\{(F_t \times G_t)(x_1,x_2), (F_t \cap G_t)(y_1,y_2)\}$
`

$(F_i \times G_i) (x_1y_1, x_2y_2) = \max\{F_i(x_1y_1), G_i(x_2y_2)\}$
$\leq \max \{\max(F_i(x_1), F_i(y_1)), \max(G_i(x_2), G_i(y_2))\}$
$\leq \max \max\{(F_i(x_1), G_i(x_2)), (F_i(y_1), G_i(y_2))\}$
$\leq \max\{ \max (F_i(x_1), G_i(x_2)), \max (F_i(y_1)), G_i(y_2))\}$
$\leq \max\{(F_i \times G_i)(x_1,x_2), (F_i \times G_i)(y_1,y_2)\}$

$(F_f \times G_f) (x_1y_1, x_2y_2) = \max\{F_f(x_1y_1), G_f(x_2y_2)\}$
$\leq \max \{\max(F_f(x_1), F_f(y_1)), \max(G_f(x_2), G_f(y_2))\}$
$\leq \max \max\{(F_f(x_1), G_f(x_2)), (F_f(y_1), G_f(y_2))\}$
$\leq \max\{ \max (F_f(x_1), G_f(x_2)), \max (F_f(y_1)), G_f(y_2))\}$

$\leq \max\{(F_f \times G_f)(x_1,x_2), (F_f \times G_f)(y_1,y_2)\}$

iii) $(F_t \times G_t)(x_1y_1, x_2y_2) = \min\{F_t(x_1y_1), G_t(x_2y_2)\}$
$\geq \min\{F_t(y_1), G_t(y_2)\}$
$\geq (F_t \times G_t)(y_1y_2)$

$(F_i \times G_i)(x_1y_1, x_2y_2) = \max\{F_i(x_1y_1), G_i(x_2y_2)\}$
$\leq \max\{F_i(y_1), G_i(y_2)\}$
$\leq (F_i \times G_i)(y_1y_2)$

$(F_f \times G_f)(x_1y_1, x_2y_2) = \max\{F_f(x_1y_1), G_f(x_2y_2)\}$
$\leq \max\{F_f(y_1), G_f(y_2)\}$
$\leq (F_f \times G_f)(y_1y_2)$

iv) $(F_t \times G_t)(x_1y_1, x_2y_2) = \min\{F_t(x_1y_1), G_t(x_2y_2)\}$
$\geq \min\{F_t(x_1), G_t(x_2)\}$
$\geq (F_t \times G_t)(x_1,x_2)$

$(F_i \times G_i)(x_1y_1, x_2y_2) = \max\{F_i(x_1y_1), G_i(x_2y_2)\}$
$\leq \max\{F_i(x_1), G_i(x_2)\}$
$\leq (F_i \times G_i)(x_1,x_2)$

$(F_f \times G_f)(x_1y_1, x_2y_2) = \max\{F_f(x_1y_1), G_f(x_2y_2)\}$
$\leq \max\{F_f(x_1), G_f(x_2)\}$
$\leq (F_f \cap G_f)(x_1,x_2)$

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